

HAT

Name Key

Unit 1 – Functions

Review

1. Determine which numbers in the set are natural numbers.

$$\sqrt{81}, 0, -13, \sqrt{17}, -4, 1, 8$$

2. Identify the rules of algebra for each step:

$$(q - 3) + 3 =$$

$$q + (-3 + 3) =$$

$$q + 0 =$$

$$q$$

3. Given $x^2 + y^2 = 9$, use the algebraic tests to determine symmetry with respect to both axes and origin.

$$\begin{aligned}x\text{-axis} \\ x^2 + (-y)^2 &= 9 \\ x^2 + y^2 &= 9 \\ \checkmark\end{aligned}$$

$$\begin{aligned}y\text{-axis} \\ (-x)^2 + y^2 &= 9 \\ x^2 + y^2 &= 9 \\ \checkmark\end{aligned}$$

$$\begin{aligned}\text{origin} \\ (-x)^2 + (-y)^2 &= 9 \\ x^2 + y^2 &= 9 \\ \checkmark\end{aligned}$$

4. Is the following equation represent y as a function of x?

$$\begin{aligned}y^2 - 5x = 2 \\ y = \pm \sqrt{2+5x} \quad \text{NO}\end{aligned}$$

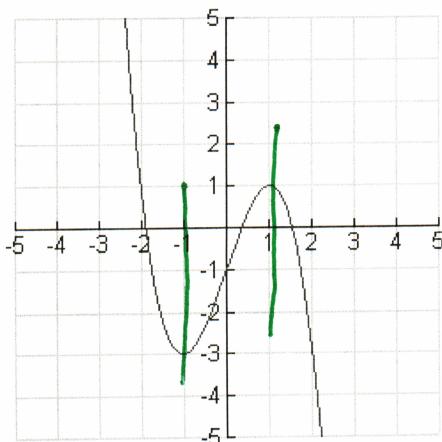
5. Find all real values of x such that $f(x) = 0$

$$\begin{aligned}\frac{3x+7}{x} \\ 3x+7 = 0 \\ x = -7/3\end{aligned}$$

6. Find the domain of the function:

$$\begin{aligned}q(t) = \frac{4t}{t+5} \\ \text{all } t \neq -5\end{aligned}$$

7. Given the following graph, determine the intervals over which the function is increasing, decreasing, or constant.



inc:
 $(-1, 1)$

dec:
 $(-\infty, -1)$
 $(1, \infty)$

8. Determine whether the function is even, odd, or neither:

$$f(x) = x^3 - 3x^2 + 5$$

neither

$$y = (-x)^3 - 3(-x)^2 + 5$$

$$y = -x^3 - 3x^2 + 5$$

No

$$\begin{aligned} &\text{odd} \\ &-y = -x^3 - 3x^2 + 5 \\ &y = x^3 + 3x^2 - 5 \\ &\text{NO} \end{aligned}$$

For problems 9-13: Given $f(x) = 4x^2 - 7x$ and $g(x) = 9 - x$ find the following:

9. $(f - g)(x)$

$$4x^2 - 6x - 9$$

12. $(f + g)(-3)$

$$4x^2 - 8x + 9$$

$$4(-3)^2 - 8(-3) + 9$$

$$36 + 24 + 9$$

$$69$$

10. $(fg)(x)$

$$\begin{aligned} &(4x^2 - 7x)(9 - x) \\ &36x^2 - 4x^3 - 63x + 7x^2 \\ &-4x^3 + 43x^2 - 63x \end{aligned}$$

13. $f(g(x))$

$$\begin{aligned} &4(9 - x)^2 - 7(9 - x) \\ &4(81 - 18x + x^2) - 63 + 7x \\ &324 - 72x + 4x^2 - 63 + 7x \\ &4x^2 - 65x + 261 \end{aligned}$$

14. Find the inverse of $f(x) = \sqrt{x+2}$, list the domain of the original and the inverse.

$$y = \sqrt{x+2} \quad \leftarrow \text{ all } \mathbb{R} \quad x \geq -2$$

$$x = \sqrt{y+2}$$

$$x^2 = y+2$$

$$x^2 - 2 = y \quad \leftarrow \text{ all } \mathbb{R} \quad x \geq 0$$

$$D: [-2, \infty)$$

$$R: [0, \infty)$$

15. Verify the following two functions are inverses of each other.

$$f(x) = 5x + 1 \quad g(x) = \frac{x-1}{5}$$

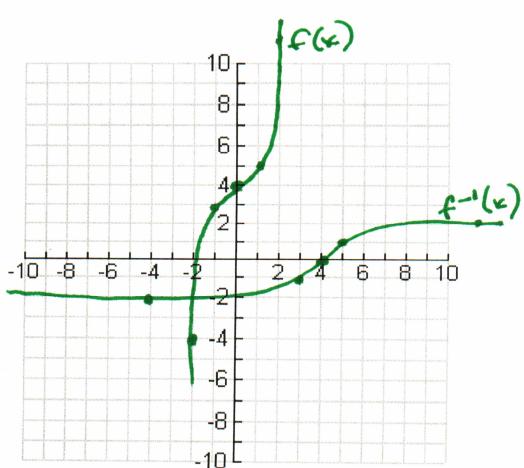
$$f(g(x)) = 5\left(\frac{x-1}{5}\right) + 1$$

$$x-1+1 = x$$

$$g(f(x)) = \frac{(5x+1)-1}{5}$$

$$= \frac{5x}{5} = x$$

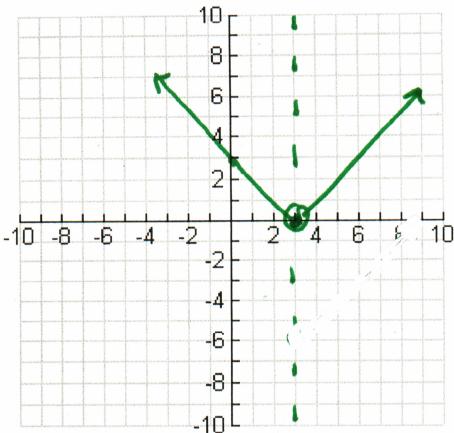
16. Graph $f(x) = x^3 + 4$ and its inverse on the same graph.



x	$f(x)$
0	4
-2	-4
-1	3
1	5
2	12

x	$f^{-1}(x)$
4	0
-4	-2
3	-1
5	1
12	2

17. Graph: $f(x) = \begin{cases} -x + 3, & \text{for } x \leq 3 \\ x - 3, & \text{for } x > 3 \end{cases}$



18. Write the equation of a square root function that has reflected on the x-axis, vertical stretch of 2, horizontal shift left 1 and a vertical shift down 4.

$$f(x) = -2\sqrt{x+1} - 4$$

19. Give the parent function, the transformation in words, and domain and range in interval notation of the following:

$$f(x) = |x|$$

$$f(x) = -|x+3| + 5$$

left 3
reflected over x axis
up 5

20. Given $f(x) = x^2 + x + 2$, simplify $\frac{f(h+1) - f(3)}{h}$, $h \neq 0$

$$\begin{aligned} f(h+1) &= (h+1)^2 + (h+1) + 2 \\ &= h^2 + 2h + 1 + h + 1 + 2 \\ &= h^2 + 3h + 4 \end{aligned}$$

$$f(3) = (3)^2 + 3 + 2$$

$$h^2 + 3h + 4 - 14$$

$$\frac{h^2 + 3h - 10}{h}$$